

# MPC for Path Following Problems of Wheeled Mobile Robots <sup>★</sup>

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**Abstract:** In this paper disturbance observer based model predictive control for path following problems of wheeled mobile robots with input disturbances is proposed. A nonlinear disturbance observer is designed to estimate the disturbances, and to compensate the influence of disturbances. Nominal model predictive control scheme with guaranteed asymptotic convergence is adopted to deal with input constraints of wheeled mobile robots. Simulation example shows that the proposed scheme can drive the wheeled mobile robots to follow a given path despite the presence of slowly varying input disturbances.

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**Keywords:** Wheeled mobile robot, path following, disturbance observer, model predictive control

## 1. INTRODUCTION

Path following problems of a wheeled mobile robot is a control task that consists of following a given path parameterized by its arc length or curvature (Jarzebowska, 2012). Compared to the tracking control problem, usually it is assumed that the wheeled mobile robot that follows the path moves forward, and the time information is not a control demand yet. That is, in general, there are no temporal specifications.

In the early days of path following control of wheeled mobile robots (WMR), scheme based on Lyapunov theory is used to design a local control law (Kanayama et al., 1990), and schemes such as feedback linearization (Luca and Benedetto, 1993; d'Andrea Novel et al., 1995), backstepping (Jiang and Nijmeijer, 1997) are used to design global (regional) control laws. Although, the existing constraints make it difficult to achieve better performance when wheeled mobile robots follow the desired (reference) path, neither input constraints nor state constraints are taken into account.

Model predictive control (MPC), also known as receding horizon control (RHC), is an advanced control strategy widely used in industrial process control. Model predictive control schemes can deal with constrained control problems, and guarantee both recursive feasibility of the involved optimization problem and stability of the closed-loop system. Receding horizon control for path following problems of wheeled mobile robot is proposed in (Faulwasser, 2012; Yu et al., 2015; Liu et al., 2017) where only nominal model is considered. Two robust MPC

schemes are proposed in (Sun et al., 2017a) for tracking unicycle robots with input constraints and bounded disturbances, where the bound is supposed to be small. A model predictive control scheme for trajectory tracking of nonholonomic mobile robots is proposed in (Nascimento et al., 2018) where a modified cost function is adopted which minimizes the distance between the robot pose and the given global trajectory coordinate. In order to reduce the computational burden, event-based model predictive tracking control of nonholonomic systems with coupled input constraints and bounded disturbances is proposed (Eqtami et al., 2013; Sun et al., 2018).

Uncertainty (disturbance or perturbation) exists widely in industrial systems, which might cause system performance degradation or even instability. To reduce the influence of uncertainty is a key point of controller design. Adaptive control method (Dixon et al., 2001; Xin et al., 2016), sliding mode control (Bloch and Drakunov, 1995; Yu et al., 2014a; Xu and Chen, 2016) and robust control (Koubaa et al., 2013; Fateh and Arab, 2015) are used to deal with uncertainties and to improve the *following accuracy* of path following problems. While the uncertainty is measurable, or the uncertainty is unmeasurable but can be estimated from the other measurable variables, the influence of the uncertainty can be compensated or eliminated (Chen et al., 2016). Disturbance observer based explicit nonlinear model predictive control is designed for the drone's flight characteristics (Liu, 2011), where neither state constraints nor input constraints of the drone are considered. By combining the disturbance observer and MPC, a disturbance rejection model predictive control scheme for tracking nonholonomic vehicles with coupled input constraints and disturbances is proposed (Sun et al., 2017b), where only the disturbance on the linear velocity is considered.

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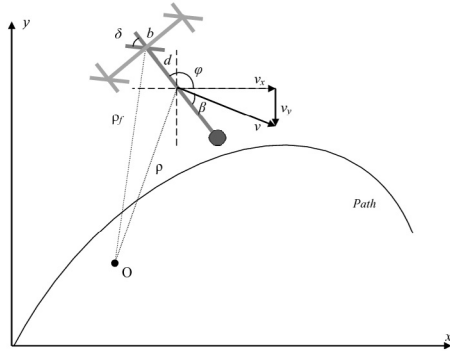


Fig. 1. Schematic diagram of the wheeled mobile robot

In this paper, disturbance observer based model predictive control scheme for path following problems of wheeled mobile robots is studied, where input constraints of the system are taken into account. Nonlinear disturbance observer is used to estimate and to compensate the influence of the input disturbances. Thus, the following accuracy of the wheeled mobile robot can be improved.

## 2. PATH FOLLOWING PROBLEM OF WHEELED MOBILE ROBOTS

A unicycle robot has a front castor and two rear driving wheels. The schematic diagram of the wheeled mobile robot is shown in Fig.1, and the symbols are described in Table 1.

Table 1. Description of symbols

Meaning	Symbol
Track between front wheels	$2b$
Vertical distance between centroid and front wheel	$d$
Radius of the wheel	$r$
Instantaneous center of robot	$O$
Distance between instantaneous center and the front wheel	$\rho_f$
Distance between instantaneous center and the centroid	$\rho$
Resultant velocity of the centroid	$v$
Side slip angle	$\beta$
Yaw angle	$\theta$
Steering angle	$\delta$
Velocity of the left wheel	$w_l$
Velocity of the right wheel	$w_f$

The motion state of the wheeled mobile robot is described by its position  $(z, y)$  and its orientation  $\theta$ , where  $(z, y)$  is the midpoint of the rear axis of the wheeled mobile robot. The kinematics equation of wheeled mobile robot is (Gu and Hu, 2006)

$$\begin{bmatrix} \dot{z} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} v \cos(\beta + \theta) \\ v \sin(\beta + \theta) \\ r(\omega_r - \omega_l)/2b \end{bmatrix} \quad (1)$$

with  $\beta = \arctan\left(\frac{\rho_f \sin \delta - d}{\rho_f \cos \delta}\right)$ , where  $v$  is the magnitude of the robot translational velocity and  $w$  is the angular velocity of  $\theta$ .

Choose steering angle  $\delta$  such that

$$\rho_f \sin \delta - d = 0,$$

i.e.,  $\beta = 0$ . Then, the kinematics equation of wheeled mobile robot is simplified as

$$\begin{bmatrix} \dot{z} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} v \cos \theta \\ v \sin \theta \\ \omega \end{bmatrix} = \begin{bmatrix} \cos \theta & 0 \\ \sin \theta & 0 \\ 0 & 1 \end{bmatrix} u \quad (2)$$

where  $u = [v \ w]^T$  is the control input,  $v = \frac{r}{2}(\omega_r + \omega_l) \cos \delta$ ,  $\omega$  is the angle velocity of the yaw angle and  $\omega = \frac{r}{2b}(\omega_r - \omega_l)$ .

Singularities play a significant role in the design and control of the robot manipulators (Donelan, 2007). A singularity is a point within the robot's workspace where the robot's Jacobin matrix loses rank. The drawbacks of singularities are (1) Loss of freedom. A drop in rank reduces the dimension of the image which represents a loss of instantaneous motion of the wheeled mobile robot. (2) Loss of control. Near a singularity, the Jacobin matrix is ill-conditioned and the control algorithm fails. Eq.(2) implies that the side slip angle  $\beta \equiv 0$  which will avoid the singularities and benefit the stability of the wheeled mobile robot.

Suppose that there is a robot moving along the reference path, then the position and orientation of the "virtual" mobile robot represent the ideal state of the wheeled mobile robot. Denote  $(z_R, y_R, \theta_R)^T$  as the reference state, and describe the kinematic model of the "virtual" wheeled mobile robot as

$$\begin{bmatrix} \dot{z}_R \\ \dot{y}_R \\ \dot{\theta}_R \end{bmatrix} = \begin{bmatrix} v_R \cos \theta_R \\ v_R \sin \theta_R \\ w_R \end{bmatrix} \quad (3)$$

where  $z_R$  and  $y_R$  denote the position of the robot,  $v_R$  is the magnitude of the robot translational velocity,  $\theta_R$  denotes the robot moving direction and  $w_R$  is the angular velocity of  $\theta_R$ .

Denote  $(x_e, y_e, \theta_e)^T$  as the error state which represents the deviation of the current position to the reference. To control the robot (2) to track the "virtual" robot (3), an error state can be defined as

$$\begin{bmatrix} z_e \\ y_e \\ \theta_e \end{bmatrix} = \underbrace{\begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{:G} \begin{bmatrix} z_R - z \\ y_R - y \\ \theta_R - \theta \end{bmatrix} \quad (4)$$

in which the linear operator  $G$  represents a coordinate transformation matrix.

Then, the error dynamic is

$$\begin{bmatrix} \dot{z}_e \\ \dot{y}_e \\ \dot{\theta}_e \end{bmatrix} = \begin{bmatrix} w y_e - v + v_R \cos \theta_e \\ -w z_e + v_R \sin \theta_e \\ w_R - w \end{bmatrix} \quad (5)$$

Denote

$$\begin{aligned} u_{1e} &= v_R \cos \theta_e - v \\ u_{2e} &= w_R - w, \end{aligned}$$

then the dynamic model of the error system is

$$\begin{aligned} \dot{z}_e &= w y_e - u_{2e} y_e + u_{1e} \\ \dot{y}_e &= -w_R z_e + u_{2e} z_e + v_R \sin \theta_e \\ \dot{\theta}_e &= u_{2e}. \end{aligned} \quad (6)$$

The error state  $(z_e, y_e, \theta_e)^T$  is close to the equilibrium while  $v$  and  $w$  approach  $v_R$  and  $w_R$ . Thus, the path

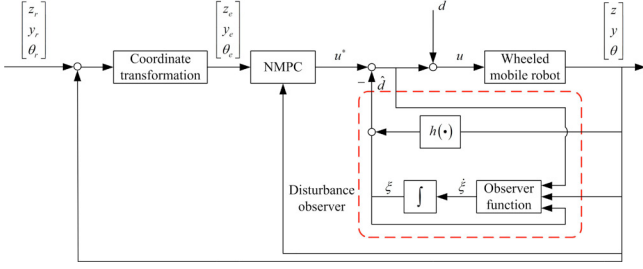


Fig. 2. Composite controller structure

following problems of the wheeled mobile robot to a given path can be reduced to the regulation problem of the error systems (5).

Denote

$$x_p = [z_e \ y_e \ \theta_e]^T$$

$$u_p = [u_{1e} \ u_{2e}]^T,$$

the system (6) can be rewritten as

$$\dot{x}_p = \beta(x_p, u_p) \quad (7)$$

where  $\beta(\cdot, \cdot)$  is twice-continuous differentiable on  $x_p$  and  $u_p$ , and  $\beta(0, 0) = 0$ .

An autonomous unicycle robot has some forward speed but zero instantaneous lateral motion, i.e., a unicycle robot is a non-holonomic system. Furthermore, the velocity  $v$  and the angular velocity  $w$  are restricted such that

$$\begin{bmatrix} v \\ w \end{bmatrix} \in \mathcal{U} \quad (8)$$

where  $\mathcal{U}$  is a compact set.

Denote  $c(s)$  as the path curvature at the given point in the reference path. For simplicity, suppose that the reference speed  $v_R$  is given, then  $w_R = c(s)v_R$ . The control objective of the wheeled mobile robot is: Given a reference path, find control laws  $[v \ w]^T \in \mathcal{U}$  such that the wheeled mobile robot follows the reference path, i.e., find suitable control laws  $[v \ w]^T$  to drive the errors  $z_e$ ,  $y_e$  and  $\theta_e$  to zero.

### 3. DISTURBANCE OBSERVER BASED ROBUST MODEL PREDICTIVE CONTROL

The wheeled mobile robot will generate a side slip phenomenon considering the influence of the ground environment when it performs a path following task. The side slip phenomenon can be treated as an input disturbance which poses a challenge to the control of the wheeled mobile robot: it may significantly degrade the following performance and may even cause instability if the influence is not taken into account in the system design.

The composite controller structure is illustrated in Fig.2. The disturbance observer provides an estimate of disturbance. An optimization problem with nominal model and tightened constraints is used in MPC to predict the system behavior and to drive the wheeled mobile robot to track the reference trajectory.

Note that disturbance observer based control is in principle a robust control scheme. Disturbance observer can estimate and compensate the system disturbance without extra sensors (extra complexities).

#### 3.1 Disturbance observer

Considering the input disturbances, system (7) can be rewritten as

$$\dot{x}_p = f(x_p) + g(x_p)(u_p + d) \quad (9)$$

where  $d = [d_v \ d_w]^T$  and

$$f(x_p) = \begin{bmatrix} c(s)v_R y_e \\ v_R \sin \theta_e - c(s)v_R z_e \\ 0 \end{bmatrix},$$

$$g(x_p) = \begin{bmatrix} 1 & -y_e \\ 0 & z_e \\ 0 & 1 \end{bmatrix}.$$

The adopted nonlinear disturbance observer is

$$\begin{aligned} \dot{\hat{d}} &= \xi + h(x_p) \\ \dot{\xi} &= -l(x_p)g(x_p)\xi \\ &\quad -l(x_p)(f(x_p) + g(x_p)u_p + g(x_p)h(x_p)) \end{aligned} \quad (10)$$

where  $\hat{d} = [d_v \ d_w]^T$  is the estimate of disturbances,  $\xi$  is the state of the nonlinear disturbance observer,  $h(\cdot)$  is a nonlinear function to be determined. The gain of the nonlinear disturbance observer satisfies that

$$l(x_p) = \frac{\partial h(x_p)}{\partial x_p} \quad (11)$$

**Assumption 1.** The derivative of the disturbance is bounded. That is, for any  $t \geq 0$ ,  $\|\dot{d}\| \leq \varepsilon$ , where  $\varepsilon > 0$  is a constant scalar.

**Theorem 1.** Suppose that the considered disturbances are input disturbances and satisfy Assumption 1. Choose  $l(x_p)$  such that  $-l(x_p)g(x_p)$  is asymptotically stable, then the estimate  $\hat{d}$  of the disturbance observer (10) can asymptotically track the input disturbances  $d$ . Furthermore,  $\|d - \hat{d}\|_\infty$  is proportional to  $\varepsilon$ .

**Proof.** Denote  $e_d := d - \hat{d}$  as the estimate error. The dynamics of the estimate error is

$$\begin{aligned} \dot{e}_d &= \dot{d} - \dot{\hat{d}} \\ &= -\xi - \frac{\partial h(x_p)}{\partial x_p} \dot{x}_p + \dot{d} \\ &= -l(x_p)h(x_p)e_d + \dot{d}. \end{aligned} \quad (12)$$

Thus, the estimate error is input-to-state stable and  $\|d - \hat{d}\|_\infty$  is proportional to  $\varepsilon$ , since  $-l(x_p)g(x_p)$  is asymptotically stable and  $\|\dot{d}\| \leq \varepsilon$  (Khalil, 2002).

**Corollary 2.** Considering the system (9), and choosing

$$l(x_e) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

the output of the disturbance observer (10) can asymptotically track the disturbance if the input disturbance satisfies Assumption 1.

**Proof.** Since  $l(x_p) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$  and  $h(x_p) = \int l(x_p)dx_p + C$  with  $C$  a constant matrix, the eigenvalues of  $-l(x_e)g(x_p) = -\begin{bmatrix} 1 & -y_e \\ 0 & 1 \end{bmatrix}$  are all located on the left half of complex plane. Due to Theorem 1, the conclusion is come to.

Note that, for simplicity,  $C = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}^T$  is chosen in the simulation example.

### 3.2 Constrained model predictive control

Suppose that  $\|e_d\| \leq \xi$ , and choose  $\mathcal{U}_0 = \mathcal{U} \ominus \xi$ , where the operator  $\ominus$  denotes the Pontryagin difference of two sets  $\mathcal{A} \subseteq \mathbb{R}^n$  and  $\mathcal{B} \subseteq \mathbb{R}^n$ .

The optimization problem is formulated as follows:

*Problem 3.*

$$\begin{aligned} & \underset{u_p(\cdot)}{\text{minimize}} && J(u_p(\cdot)) \\ & \text{subject to} && \\ & \dot{x}_p(\tau, x_p(t)) = f(x_p(\tau, x_p(t))) + g(x_p(\tau, x_p(t)))u(\tau), \\ & x_p(t, x_p(t)) = x_p(t), \\ & u(\tau) \in \mathcal{U}_0, \tau \in [t, t + T_p], \\ & x_p(t + T_p, x(t)) \in \Omega, \end{aligned}$$

where

$$J(u(\cdot)) := \|x_p(t + T_p, x_p(t))\|_P^2 + \int_t^{t+T_p} (\|x_p(\tau, x_p(t))\|_Q^2 + \|u_p(\tau)\|_R^2) d\tau$$

is the cost functional,  $T_p$  is the prediction horizon,  $Q \in \mathbb{R}^{n_3 \times n_3}$  and  $R \in \mathbb{R}^{n_2 \times n_2}$  are positive definite state and input weighting matrices. Note that  $\|v\|_Q = \sqrt{v^T Q v}$  with  $Q \in \mathbb{R}^{n \times n}$  and  $Q > 0$  for a vector  $v \in \mathbb{R}^n$ , and  $u(\tau) = \begin{bmatrix} v_R \cos \theta_e(\tau) - u_{1e}(\tau) \\ c(s)v_R - u_{2e}(\tau) \end{bmatrix}$ .

The positive definite matrix  $P \in \mathbb{R}^{n_3 \times n_3}$  is the terminal penalty matrix, and  $E(x) := \|x\|_P^2$  is the terminal penalty function. The terminal set  $\Omega := \{x \in \mathbb{R}^{n_3} \mid x^T P x \leq \alpha\}$  is a level set of the terminal penalty function. The term  $x_p(\cdot, x_p(t))$  represents the predicted state trajectory starting from the initial state  $x_p(t)$  under the control  $u(\cdot)$ . In order to guarantee feasibility and convergence,  $P$  and  $\Omega$  have to satisfy terminal conditions, see (Chen and Allgöwer, 1998) and (Mayne et al., 2000).

*Remark 4.* Only nominal model is adopted to predict the system dynamics in Problem 3.

*Remark 5.* Similar to (Yu et al., 2015), the initial state of the reference path can be chosen as a new determined variable in Problem 3.

Since  $e_d$  keeps small and only input constraints are considered here, similar to (Yu et al., 2014b), both recursive feasibility of the optimization problem and asymptotic convergence of the system (9) to the origin can be guaranteed. Thus, the wheeled mobile robot can follow asymptotically the reference path.

### 3.3 Composite control law

The overall control input is

$$u(t) = u(t)^* - \hat{d}(t) \quad (13)$$

where  $u(t)^*$  is the first segment of control input obtained by the online optimization of model predictive control.

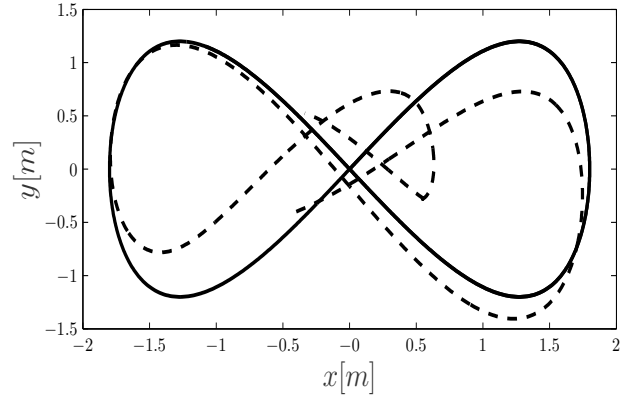


Fig. 3. Desired eight-shaped curve (solid line) and the actual trajectory (dashed line) of the wheeled mobile robot: with MPC

## 4. SIMULATIONS

In order to verify the effectiveness of the proposed scheme, a simulation experiment is carried out in Matlab. The physical parameters of the robot is as follows:  $r = 8\text{cm}$ ,  $b = 23\text{cm}$  and  $d = 75\text{cm}$ .

The speed constraint is  $-1 \leq v \leq 3\text{m/s}$ , the angular speed constraint is  $-3.5 \leq \omega \leq 3.5\text{rad/s}$ , the magnitude of the translational velocity of the “virtual” wheeled mobile robot is  $v_R = 0.7$ .

The cost function is

$$F(x_p, u_p) = x_p^T Q x_p + u_p^T R u_p \quad (14)$$

where the weighting matrices are  $Q = 0.4I_3$  and  $R = 0.5I_2$  with  $I_j \in \mathbb{R}^{j \times j}$  identity matrix.

Consider the input disturbances

$$d_v = \begin{cases} 0.5(1 - e^{-0.05(t-12)}), & t \in [12, \infty) \\ 0, & t \in [0, 12) \end{cases} \quad (15)$$

and  $d_w = 0$ .

The prediction horizon is 10 and the sampling time is 0.02s. The terminal penalty matrix is

$$P = \begin{bmatrix} 4.3483 & 0 & 0 \\ 0 & 4.7593 & 4.3374 \\ 0 & 4.3374 & 23.3629 \end{bmatrix} \quad (16)$$

and the terminal set is  $\Omega = \{x_p \in \mathbb{R}^3 \mid x_p^T P x_p \leq 10\}$ .

### 4.1 Eight-shaped curve tracking

The reference trajectory is an eight-shaped trajectory

$$\begin{aligned} z_R &= 1.8 \sin(\theta_R) \\ y_R &= 1.2 \sin(2\theta_R). \end{aligned} \quad (17)$$

The initial position of the mobile robot is  $(z_0, y_0, \theta_0)^T = (-0.4, -0.4, \pi/2)^T$ . The actual trajectory of the wheeled mobile robot is shown in Fig. 3 as dashed line, and the reference trajectory is shown as solid line, where model predictive control for path following problems (Yu et al., 2015; Liu et al., 2017) is adopted. It shows that the actual trajectory of the wheeled mobile robot deviates from the reference trajectory and the deviation is increasing while the input disturbances (15) acts on. In order to reduce the influence of input disturbances and improve

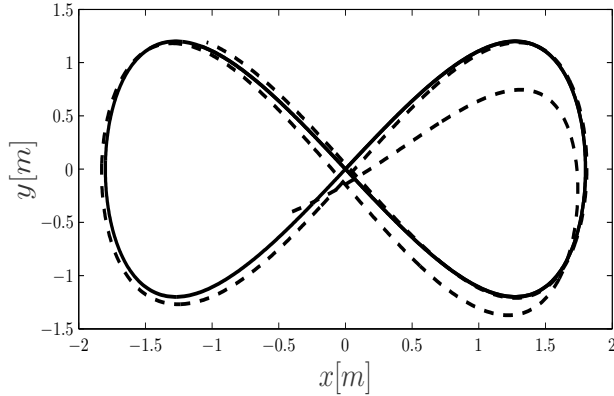


Fig. 4. Desired eight-shaped curve (solid line) and the actual trajectory (dashed line) of the wheeled mobile robot: with disturbance observer based MPC

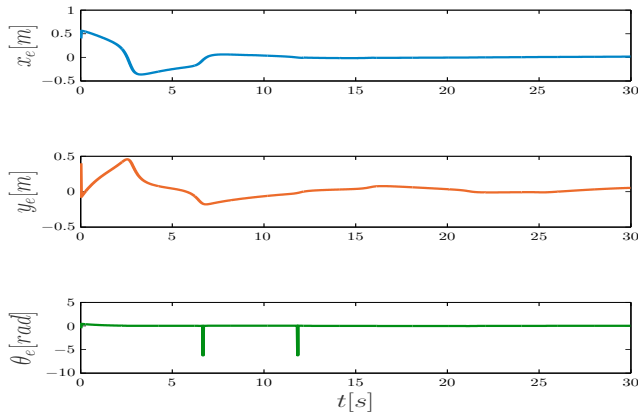


Fig. 5. Eight-shaped curve: error curve with disturbance observer based MPC

the following accuracy, disturbance observer based MPC is adopted. Fig.4 shows the comparison between the actual trajectory and the desired trajectory of the wheeled mobile robot. Although there exist slowly varying and bounded disturbances, the wheeled mobile robot with disturbance observer based MPC can follow the desired path. Fig.5 shows the error curve of the wheeled mobile robot. Since the disturbance observer can estimate and compensate the input disturbances, the error will asymptotically converge to zero. The evolution of the control input of the wheeled mobile robot is shown in Fig.6, which satisfies the input constraints. Fig. 7 shows the output of the disturbance observer. On one hand, the disturbance observer can estimate the real disturbances; on the other hand, the estimate error does not asymptotically converge to zero since the error system (7) is state-dependent.

## 5. CONCLUSION

Disturbance observer based model predictive control for path following problems of wheeled mobile robots was proposed in this paper. While there is no disturbance at all, model predictive control can guarantee the satisfaction of input constraints, and drive the wheeled mobile robot to the desired path. While there are input disturbances, in particular, slowly varying disturbances, nonlinear disturbance observer can estimate the disturbances and compensate the influence of it through a “feedback”. Simulation

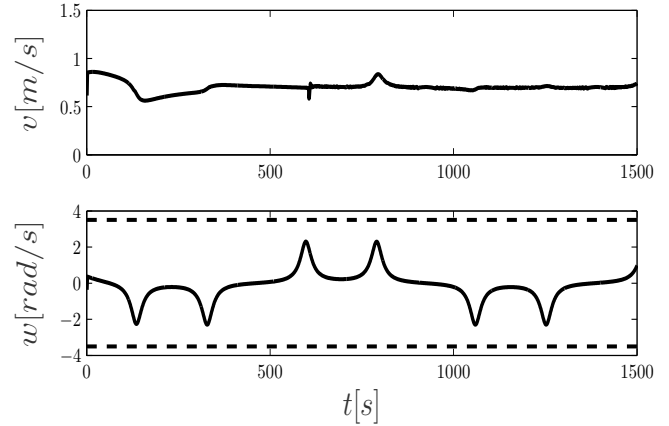


Fig. 6. Eight-shaped curve: Control curve with disturbance observer based MPC

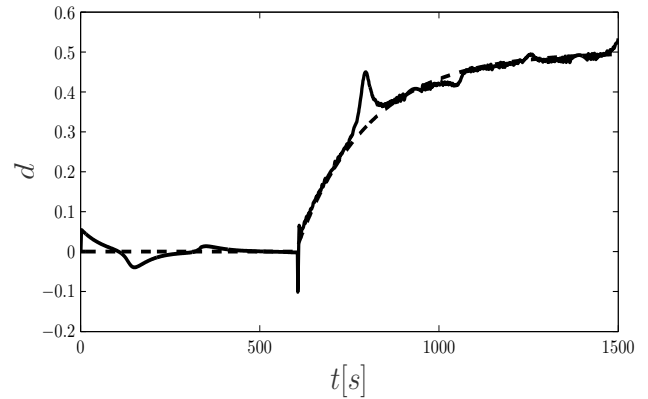


Fig. 7. Eight-shaped curve: the estimated disturbances

result showed that the proposed control strategy can guarantee the convergence of the wheeled mobile robot to the reference path with respect to input disturbances.

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